Bridging work for A level Mathematics

Maths Department

2025

Name____________________

Waseley Hills
High School

A Level Mathematics – Bridging Work

Waseley Hills High School

In order to achieve in A Level Mathematics it is **vital** that you have a secure knowledge of GCSE Mathematics content. In particular, you must be **fluent** in the following topics:

- Factorising expressions.
- Completing the square.
- Solving quadratic equations by factorisation, by completing the square and by the quadratic formula.
- Sketching quadratic graphs.
- Solving linear simultaneous equations using both elimination and substitution.
- Solving linear and quadratic simultaneous equations.
- Linear inequalities.
- Quadratic inequalities.
- Straight line graphs.
- Parallel and perpendicular lines.
- Pythagoras' theorem.
- Proportion.

We expect that most students will already be confident in the vast majority of these topics.

It is essential that all students spend a significant amount of time practising these topics at regular intervals between the end of Year 11 and the start of Year 12.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

Mark your work using the 'Answers' attached at the end of every topic, checking that you have understood.
If you find that you have made mistakes, **identify** and **correct** these. If you cannot do this, reread the 'Examples' for that specific topic, to ensure that you have not misunderstood a concept. If you still do not understand something and cannot understand why, you are welcome to email the Head of Maths, Dr Parsons, at sparsons@waseleyhills.worcs.sch.uk for further resources.
Complete and mark the 'Extend' questions to make sure that you do have an excellent understanding.

Please bring all of your **completed and marked** bridging work to your first maths lesson where it will be checked by your maths teacher. We expect you to complete the questions on lined or squared paper, showing a **full method** and **working out**.

There will be a baseline assessment covering these topics in the first weeks of Year 12. It is expected that all A Level Mathematics students will demonstrate an excellent understanding of all topics in this assessment.

Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** ($2x - 5$) is a factor of both terms

Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$ So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) **3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** ($x - 7$) is a factor of both terms
For the denominator: $b = 9$, $ac = 18$ So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) **7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** ($x + 3$) is a factor of both terms
So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \frac{x-7}{2x+3}$	**10** ($x + 3$) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$

b $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

> **Hint**
>
> Take the highest common factor

2 Factorise

a $x^2 + 7x + 12$

b $x^2 + 5x - 14$

c $x^2 - 11x + 30$

d $x^2 - 5x - 24$

e $x^2 - 7x - 18$

f $x^2 + x - 20$

g $x^2 - 3x - 40$

h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$

b $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$

b $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$

d $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$

f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2 + 4x}{x^2 - x}$

b $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$

c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$

d $\dfrac{x^2 - 5x}{x^2 - 25}$

e $\dfrac{x^2 - x - 12}{x^2 - 4x}$

f $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$

b $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$

d $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$

c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$

c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$

e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$

g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$

c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$

c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$

e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$

c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$

e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 **a** $(x + 2)^2 - 1$ **b** $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$ **d** $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$ **f** $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$ **d** $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 **a** $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ **b** $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ **d** $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x + 3)^2 + 3$

Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.
So $(3x + 4) = 0$ or $(3x - 4) = 0$	**2** When two values multiply to make zero, at least one of the values must be zero.
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (−8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	**2** Rewrite the b term ($-5x$) using these two factors.
$2x(x - 4) + 3(x - 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x - 4)(2x + 3) = 0$	**4** $(x - 4)$ is a factor of both terms.
So $(x - 4) = 0$ or $(2x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
$x = 4$ or $x = -\frac{3}{2}$	**6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of

Solving quadratic equations by completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	**2** Simplify.
$(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	**5** Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x-\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x-\frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x-\frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.

	(continued on next page)
$2\left(x-\frac{7}{4}\right)^2=\frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x-\frac{7}{4}\right)^2=\frac{17}{16}$	**6** Divide both sides by 2.
$x-\frac{7}{4}=\pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x=\pm\frac{\sqrt{17}}{4}+\frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x=\frac{7}{4}-\frac{\sqrt{17}}{4}$ or $x=\frac{7}{4}+\frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of

Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify *a*, *b* and *c* and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over 2*a*, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

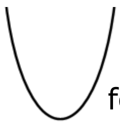
c $x = -1\frac{2}{3}$ or $x = 2$

Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.



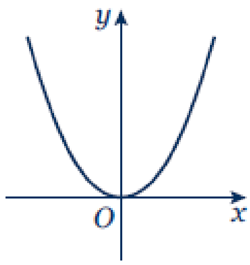
for $a >$

for $a <$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

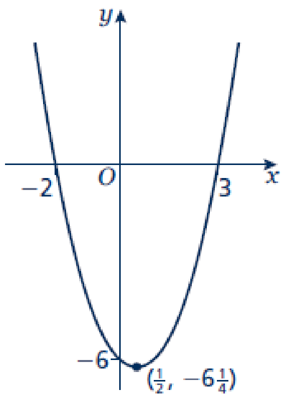
Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	**2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape: 

(continued on next page)

$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	**6** To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

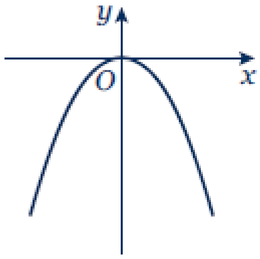
5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

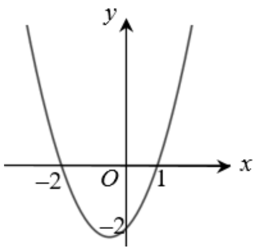
6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

1



2 **a**



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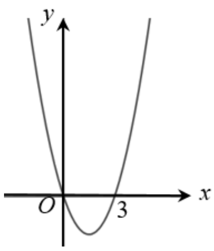

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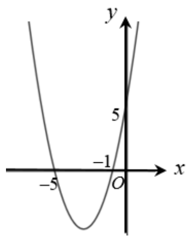
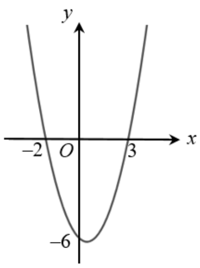


3 **a**



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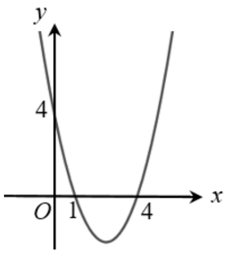

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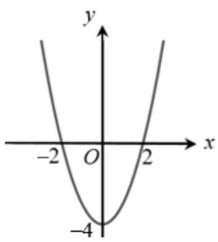

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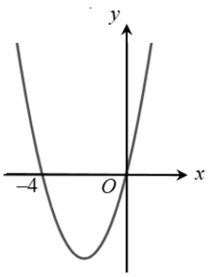

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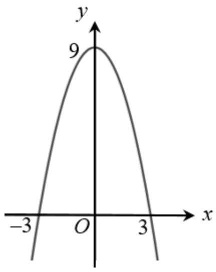

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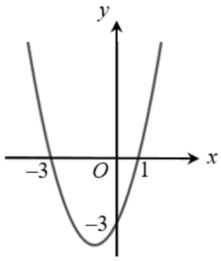

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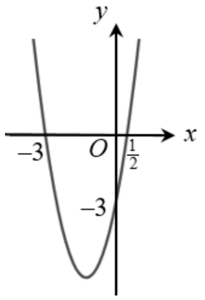
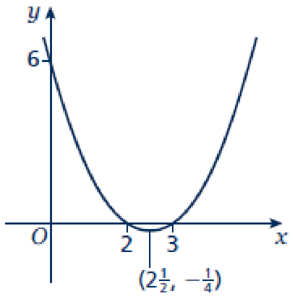


5 **a**



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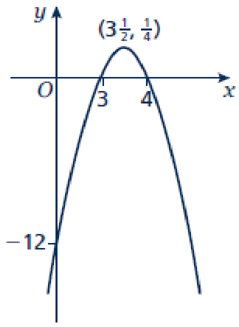
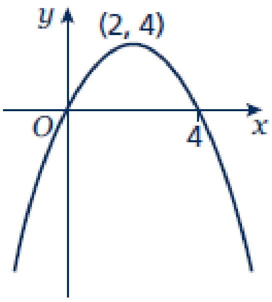


c



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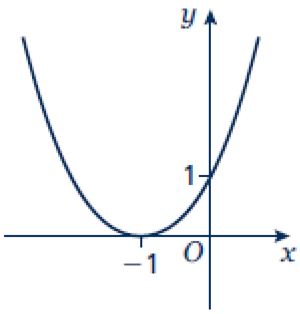


Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+\quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify. **4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = –2$

3 $x = 2, y = –5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = –1$

6 $x = –2, y = 5$

7 $x = 9, y = 5$

8 $x = –2, y = –7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = –4, y = 5$

12 $x = –2, y = –5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = –2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$x = -\dfrac{9}{5},\ y = -\dfrac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \dfrac{16}{5},\ y = \dfrac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\dfrac{8}{3},\ y = -\dfrac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \dfrac{5}{2},\ y = 6$

$x = 3, y = 5$

11 $x = \dfrac{1+\sqrt{5}}{2},\ y = \dfrac{-1+\sqrt{5}}{2}$

$x = \dfrac{1-\sqrt{5}}{2},\ y = \dfrac{-1-\sqrt{5}}{2}$

12 $x = \dfrac{-1+\sqrt{7}}{2},\ y = \dfrac{3+\sqrt{7}}{2}$

$x = \dfrac{-1-\sqrt{7}}{2},\ y = \dfrac{3-\sqrt{7}}{2}$

Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \le 4x < 16$

$-8 \le 4x < 16$ $-2 \le x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$ $\frac{4}{5} \le x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \ge -8$

$2 - 5x \ge -8$ $-5x \ge -10$ $x \le 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1	**a**	$x > 4$	**b**	$x \leq 2$	**c**	$x \leq -1$
	d	$x > -\frac{7}{2}$	**e**	$x \geq 10$	**f**	$x < -15$
2	**a**	$x < -20$	**b**	$x \leq 3.5$	**c**	$x < 4$
3	**a**	$x \leq -4$	**b**	$-1 \leq x < 5$	**c**	$x \leq 1$
	d	$x < -3$	**e**	$x > 2$	**f**	$x \leq -6$
4	**a**	$t < \frac{5}{2}$	**b**	$n \geq \frac{7}{5}$		
5	**a**	$x < -6$	**b**	$x < \frac{3}{2}$		

6 $x > 5$ (which also satisfies $x > 3$)

Quadratic inequalities

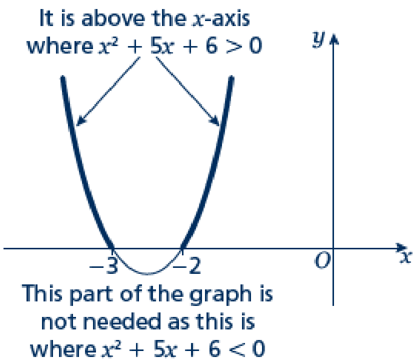
A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

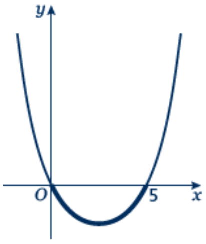
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

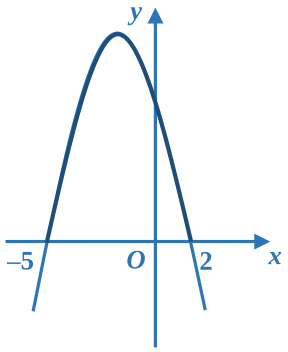
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers

1 $-7 \le x \le 4$

2 $x \le -2$ or $x \ge 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \le x \le 4$

6 $-3 \le x \le 2$

7 $2 < x < 2\frac{1}{2}$

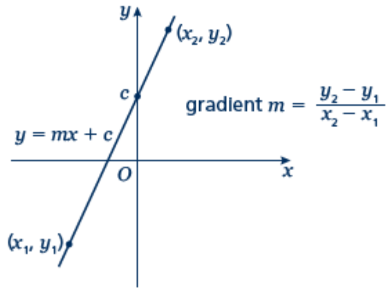
8 $x \le -\frac{3}{2}$ or $x \ge \frac{5}{3}$

Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	**3** Simplify and solve the equation.
$y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7

b gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4

d gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17)

b (0, 6), (–4, 8)

c (–1, –7), (5, 23)

d (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

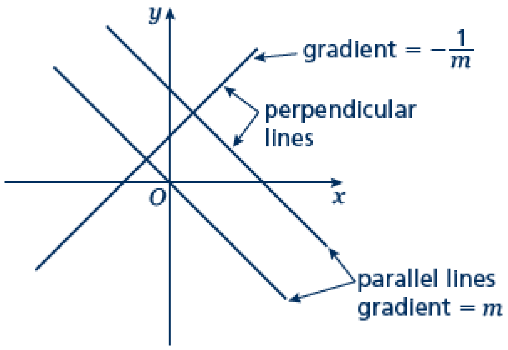
7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3. The line intercepts the axes at $(0, 3)$ and $(2, 0)$. Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates into the equation $y = 2x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. **2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2)
b $y = 3 - 2x$ (1, 3)
c $2x + 4y + 3 = 0$ (6, –3)
d $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

Hint
If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0)
b $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)
c $x - 4y - 4 = 0$ (5, 15)
d $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points *A* and *B* with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point *C* with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

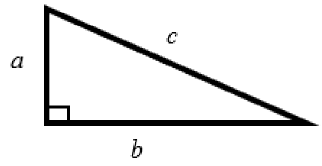
6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Pythagoras’ theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

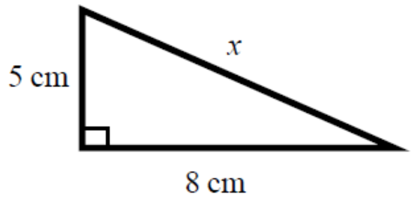
Key points

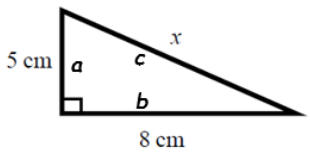
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras’ theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



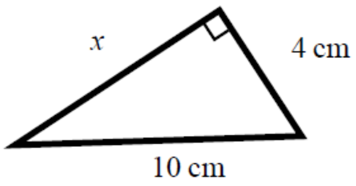
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$  	**1** Always start by stating the formula for Pythagoras’ theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$ $x = 9.433\ 981\ 13...$ $x = 9.43$ cm	**2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root. **4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length x.
Give your answer in surd form.

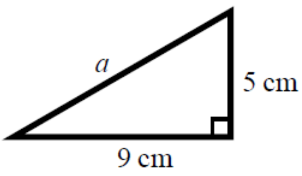


$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of a, b and c into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

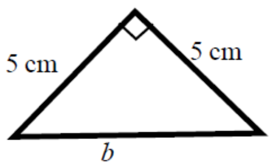
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

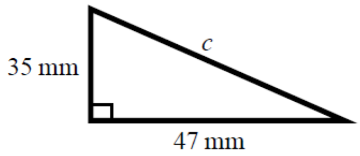
a



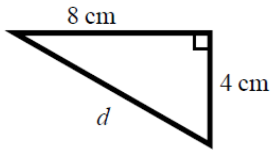
b



c

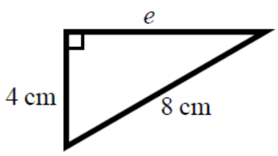


d

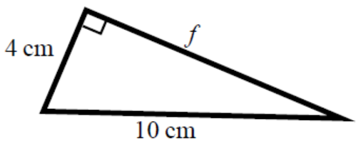


2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

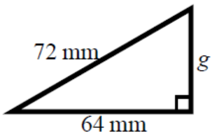
a



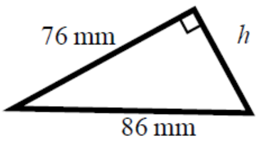
b



c

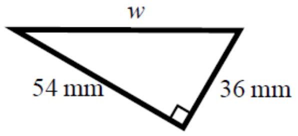


d

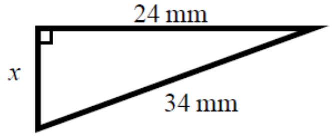


3 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

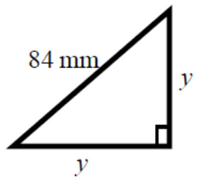
a



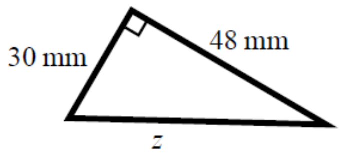
b



c



d



4 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

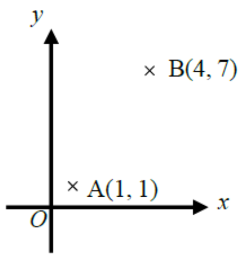
> **Hint**
>
> Draw a sketch of the

Extend

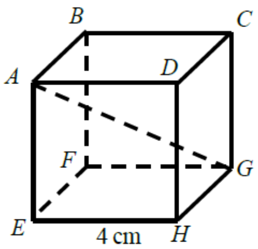
5 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given

6 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



7 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



Answers

1 **a** 10.3 cm **b** 7.07 cm

c 58.6 mm **d** 8.94 cm

2 **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm

c $8\sqrt{17}$ mm **d** $18\sqrt{5}$ mm

3 **a** $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm

c $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

4 95.3 mm

5 64.0 km

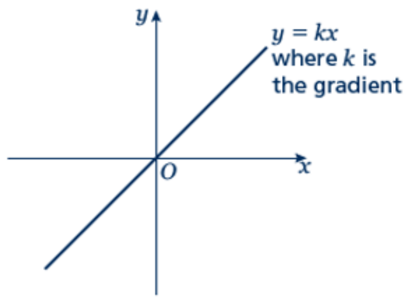
6 $3\sqrt{5}$ units

7 $4\sqrt{3}$ cm

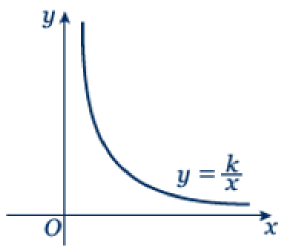
Proportion

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points



- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as $y \propto x$. If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y, the graph is a straight line passing through the origin.



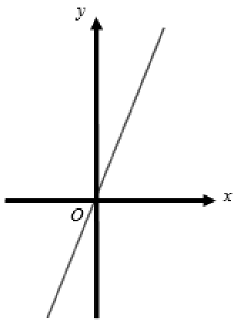
- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as $y \propto \frac{1}{x}$.

  If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$

Examples

Example 1 y is directly proportional to x.
When $y = 16$, $x = 5$.
a Find x when $y = 30$.
b Sketch the graph of the formula.

a $y \propto x$	**1** Write y is directly proportional to x, using the symbol $\propto$.
$y = kx$	**2** Write the equation using k.
$16 = k \times 5$	**3** Substitute $y = 16$ and $x = 5$ into $y = kx$.
$k = 3.2$	**4** Solve the equation to find k.
$y = 3.2x$	**5** Substitute the value of k back into the equation $y = kx$.
When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	**6** Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.

b 	**7** The graph of $y = 3.2x$ is a straight line passing through (0, 0) with a gradient of 3.2.

Example 2 y is directly proportional to x^2.
When $x = 3$, $y = 45$.
a Find y when $x = 5$.
b Find x when $y = 20$.

a $y \propto x^2$	**1** Write y is directly proportional to x^2, using the symbol $\propto$.
$y = kx^2$ $45 = k \times 3^2$	**2** Write the equation using k. **3** Substitute $y = 45$ and $x = 3$ into $y = kx^2$.
$k = 5$ $y = 5x^2$	**4** Solve the equation to find k. **5** Substitute the value of k back into the equation $y = kx^2$.
When $x = 5$, $y = 5 \times 5^2$ $y = 125$	**6** Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$.
b $20 = 5 \times x^2$ $x^2 = 4$ $x = \pm 2$	**7** Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 4$.

Example 3 P is inversely proportional to Q.
When $P = 100$, $Q = 10$.
Find Q when $P = 20$.

$P \propto \frac{1}{Q}$	**1** Write P is inversely proportional to Q, using the symbol $\propto$.
$P = \frac{k}{Q}$	**2** Write the equation using k.
$100 = \frac{k}{10}$	**3** Substitute $P = 100$ and $Q = 10$.
$k = 1000$	**4** Solve the equation to find k.
$P = \frac{1000}{Q}$	**5** Substitute the value of k into $P = \frac{k}{Q}$
$20 = \frac{1000}{Q}$	**6** Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.

$Q = \frac{1000}{20} = 50$	

Practice

Hint

Substitute the values given for *P* and *h* into the formula to

1 Paul gets paid an hourly rate. The amount of pay (£*P*) is directly proportional to the number of hours (*h*) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 x is directly proportional to y.
$x = 35$ when $y = 5$.

a Find a formula for x in terms of y.

b Sketch the graph of the formula.

c Find x when $y = 13$.

d Find y when $x = 63$.

3 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

a Find a formula for Q in terms of Z.

b Sketch the graph of the formula.

c Find Q when $Z = 5$.

d Find Z when $Q = 300$.

4 y is directly proportional to the square of x.
$x = 2$ when $y = 10$.

a Find a formula for y in terms of x.

b Sketch the graph of the formula.

c Find x when $y = 90$.

5 B is directly proportional to the square root of C.
$C = 25$ when $B = 10$.

a Find B when $C = 64$.

b Find C when $B = 20$.

6 C is directly proportional to D.
$C = 100$ when $D = 150$.
Find C when $D = 450$.

7 y is directly proportional to x.
$x = 27$ when $y = 9$.
Find x when $y = 3.7$.

8 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

Extend

9 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

10 a is inversely proportional to b.
$a = 5$ when $b = 20$.

a Find a when $b = 50$.

b Find b when $a = 10$.

11 v is inversely proportional to w.
$w = 4$ when $v = 20$.

a Find a formula for v in terms of w.

b Sketch the graph of the formula.

c Find w when $v = 2$.

12 L is inversely proportional to W.
$L = 12$ when $W = 3$.
Find W when $L = 6$.

13 s is inversely proportional to t.
$s = 6$ when $t = 12$.

a Find s when $t = 3$.

b Find t when $s = 18$.

14 y is inversely proportional to x^2.
$y = 4$ when $x = 2$.
Find y when $x = 4$.

15 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

16 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

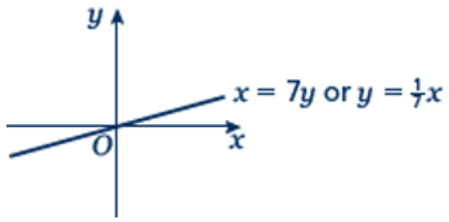
a Find a when $b = 2$.

b Find b when $a = 2$.

Answers

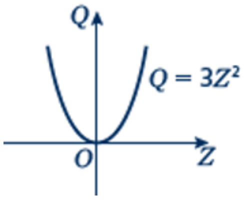
1 £77

2 **a** $x = 7y$ **b**



c 91 **d** 9

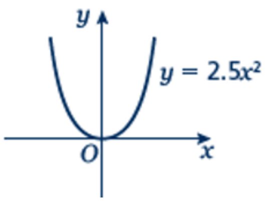
3 **a** $Q = 3Z^2$ **b**



c 75 **d** ±10

4 **a** $y = 2.5x^2$ **b**

c ±6



5 **a** 16 **b** 100

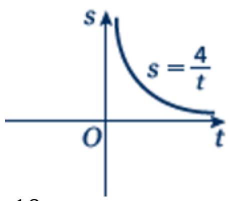
6 300

7 11.1

8 5

9 **a** $s = \frac{4}{t}$ **b**

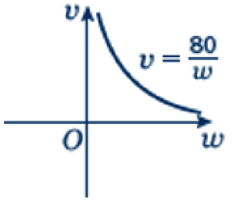
c 4



10 **a** 2 **b** 10

11 **a** $v = \frac{80}{w}$ **b**

c 40



12 6

13 **a** 24 **b** 4

14 1

15 1

16 **a** 0.1 **b** 0.1